PHY2021 Electromagnetism I
Week 9 Problems: Magnetic Potentials, Faraday and Lenz Laws

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1. (a) Give an expression relating the magnetic vector potential $\boldsymbol{A}$ to the magnetic induction $\boldsymbol{B}$.

   (b) Show mathematically that a given magnetic induction $\boldsymbol{B}$ is not uniquely related to a magnetic vector potential $\boldsymbol{A}$.

   (c) Show that, in cylindrical coordinates,

   $$\boldsymbol{A} = \frac{B}{2}\hat{\boldsymbol{z}} \times \boldsymbol{r} \tag{1}$$

   is a possible vector potential for the uniform magnetic induction $B\hat{\boldsymbol{z}}$.

   (d) In magnetostatics, derive an expression for the current density $\boldsymbol{J}$ that would produce the vector potential $\boldsymbol{A} = k\hat{\boldsymbol{\theta}}$, expressed in cylindrical coordinates where k is a constant.

   (e) Show that this vector potential satisfies the Coulomb gauge.

2. A long straight wire carries a constant current $\boldsymbol{I} = I\hat{\boldsymbol{z}}$.

   (a) Using the integral form of Ampere's Law, derive an expression in cylindrical coordinates for the magnetic induction $\boldsymbol{B}$ outside the wire.

   (b) Show that in cylindrical coordinates

   $$\boldsymbol{A} = \left(-\frac{\mu_0 I}{2\pi}\ln r + c\right)\hat{\boldsymbol{z}}, \tag{2}$$

   is a possible vector potential for the magnetic induction outside the wire, where c is a constant.

3. (a) Beginning from the differential form of the Faraday-Lenz law, derive the integral form.

   (b) A uniform magnetic field $\boldsymbol{B}$ is aligned with the z axis. A circular conducting loop of radius R centred on the origin rotates with angular velocity ω. At time $t = 0$ the loop lies in the $x - y$ plane, and the loop rotates about the x axis.
   Derive an expression for the magnetic flux Ψ_m passing through the loop as a function of time.

   (c) Derive an expression for the emf induced in the loop as a function of time.