PHY2021 Electromagnetism I
Week 4 Problems: Gauss's Law and Multipole Moments

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The volume and surface elements in cylindrical coordinates are [1]

$$dS = rd\phi dz \qquad\qquad dV = rdrd\phi dz.$$

[1] https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

1. (a) Evaluate the indefinate integral [2]

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx$$

[2] Consider the substitution $x = a\sinh u$, remembering that $\cosh^2\theta - \sinh^2\theta = 1$.

(b) Evaluate the integral [3]

$$\int \frac{x}{(a^2 + x^2)^{3/2}} dx$$

[3] Hint: The substitution $u = a^2 + x^2$ might simplify the problem.

2. Given an electric field

$$\boldsymbol{E} = kf(r)\hat{\boldsymbol{r}},$$

where k is a constant and $f(r)$ is some function of radial position r, find an expression for the charge density associated with the electric field.

3. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder, of radius a and a uniform surface charge density σ on the outer cylindrical shell, which has radius b. The surface charge is negative and is of the correct magnitude that the cable as a whole is electrically neutral.
Find the electric field in each of the three regions:
(i) $r < a$,
(ii) $a < r < b$,
(iii) $r > b$.
Plot $|\boldsymbol{E}|$ as a function of r.

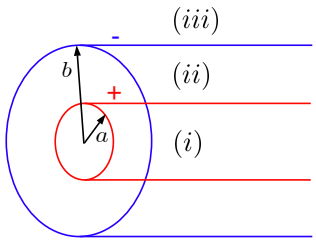

Figure 1: The situation considered in question 3.

4. Consider a cylinder of radius R and length L centered at the origin, laying along the z axis, with a charge density

$$\rho = \rho_0 \sin\left(\frac{\pi z}{L}\right).$$

(a) Plot the charge density as a function of z.

(b) Find the dipole moment of the cylinder.