PHY2021 Electromagnetism I
Week 2 Problems: Electrostatics

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These questions are not taken from exam papers.

1. Sketch the vector field
$$v = \frac{\hat{r}}{r^2},$$
and calculate its divergence, $\nabla \cdot v$ using spherical coordinates.

2. (a) Find the electric field at a distance z above the mid-point of two charges separated by a distance d. See Figure 1.

   (b) Expand the field for large z (i.e. $z \gg d$). What happens? You will need to use the following expansion
$$(1+x)^\alpha \approx 1 + \alpha x,$$
which is valid when $x \ll 1$.

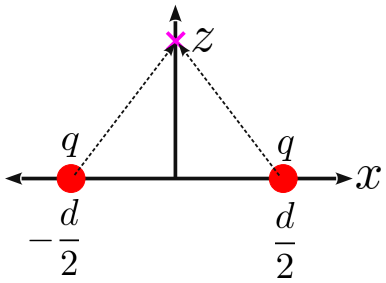

Figure 1: Schematic of the two charge problem in question 2.

3. The potential energy of an electron with itself is
$$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

   (a) What happens as $r \to 0$? [1].

   (b) To determine the radius at which classical electromagnetism breaks down, r_0, solve $U = mc^2$ for r and calculate the numerical value of r_0. Compare this with the radius of the electron. (Here we are simply requiring that electromagnetism is consistent with special relativity. The rest energy of the electron should be equal to its mass energy.)

   (c) In reality, quantum effects become important long before this, at length scales $r_{qm} \sim \hbar/mc$. Find the numerical value of the ratio r_0/r_{qm} [2].

[1] This is a bit of a delicate subject in electromagnetism and quantum electrodynamics. Many methods have been developed for dealing with this problem, for example 'mass renormalization'. A nice introduction to this problem in modern physics is given by John Baez in "Struggles with the Continuum" (`https://arxiv.org/pdf/1609.01421.pdf`)

[2] This is the fine-structure constant, a fundamental physical constant determining the strength of the electromagnetic iteration between elementary particles.