PHY2021 Electromagnetism I
Week 1 Problems: Introduction to Fields

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October 12, 2020

These problems are not taken from exam papers. The aim is to get you thinking about vectors and playing around with some simple electrostatics problems.

1. Show that the dot product of two vectors, $A \cdot B$, is invariant under a rotation of the coordinate system about the z axis.
   The rotation matrix in three-dimensions is

$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

   (How might you have guessed the dot product would be invariant?)

2. (a) Calculate $\nabla(r^2)$, where $r = x\hat{x} + y\hat{y} + z\hat{z}$ is a position vector and $r = |r|$.

   (b) Calculate $\nabla(1/r)$.

   (c) Show that $\nabla(r^n) = nr^{n-2}r$.